A NEW PERSPECTIVE ON TURBULENT GALACTIC MAGNETIC FIELDS

The unification of linear polarisation decomposition techniques

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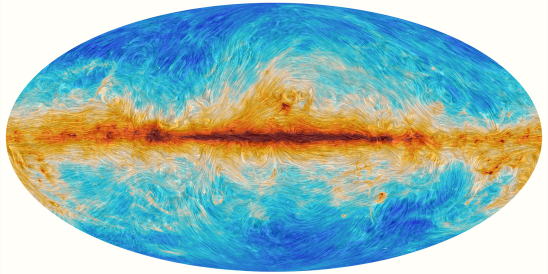
MAGNETIC FIELDS IN THE MILKY WAY



353 GHz polarised dust emission - ESA and the Planck Collaboration

- Fluctuations in magnetic fields
  - Large-scale coherent fluctuations aligned with the spiral arms (ordered field)
  - Small-scale turbulent fluctuations (random field)
- Turbulence transfers energy from large to smaller scales
  - Multiple energy-injection scales?
  - Most important contributions?
- How can we quantify these fluctuations?

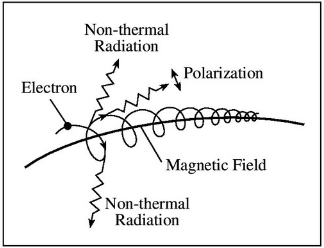
SYNCHROTRON EMISSION

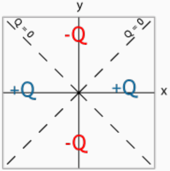
Emission of relativistic electrons around magnetic field lines

$$\frac{d}{d\tau}(m\gamma\mathbf{v}) = q\left(\frac{\mathbf{v}}{c} \times \mathbf{B}\right)$$

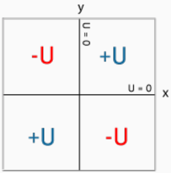
$$L_\nu \propto \nu^{-\alpha}$$

- Linear polarisation
- Luminosity depends on the frequency emission
- Polarisation describe by the two pseudo-vectors Stokes Q and U.





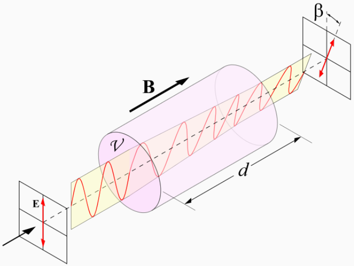
Sign of Q



Sign of U

DIFFUSE SYNCHROTRON EMISSION

Synchrotron radiation carries the imprint of the magnetic field at the point of origin and along the propagation path.



Linear polarisation vector	$\mathrm{P} = \mathrm{Q} + i\mathrm{U}$
Polarisation intensity	$\lvert\mathbf{P}\rvert = \sqrt{\mathrm{Q}^2 + \mathrm{U}^2}$
Angle of polarisation	$\theta = (1/2)\tan^{-1}(\mathrm{U}/\mathrm{Q})$
Faraday rotation	$\theta = \theta_0 + \phi\lambda^2$
Faraday depth	$\phi = 0.81 \int_{\text{source}}^{\text{observer}} n_e \mathbf{B} \cdot dl$

ROTATION MEASURE

Stil, Taylor & Sunstrum 2011

THE ASTROPHYSICAL JOURNAL, 726:4 (14pp), 2011 January 1

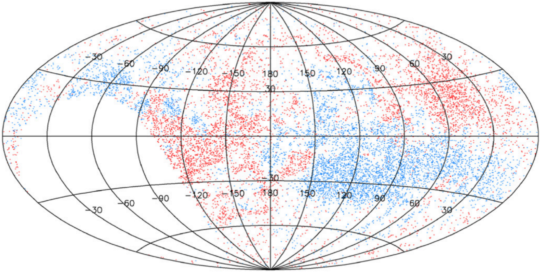
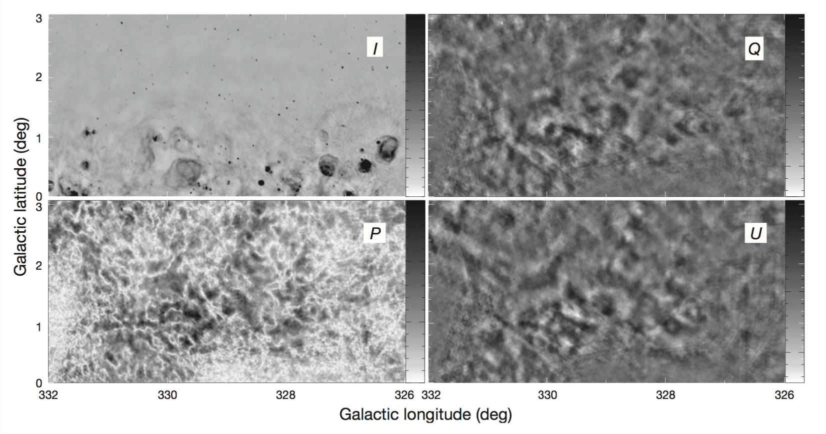


Figure 4. RM structure on the sky centered at $(l, b) = (180°, 0°)$, leaving out data with $|\mathrm{RM}| < 25$ rad m^{-2}, with positive RM in red and negative RM in blue, and no scaling of the circles according to RM amplitude. This representation is analogous to showing a the lowest contour in a radio map at the level of a few sigma. It visualizes RM structure and boundaries where the sign of RM changes. Regions with a high RM amplitude appear more densely sampled. Note the arc-like structures in RM on angular scales of tens of degrees that add a few hundred rad^2 m^{-4} to the SF amplitude on large angular scales.

Gaensler et al. 2011



LIMITS OF INTERPRETATION

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    - A smooth distribution of intervening polarized emission
    - A smooth screen of foreground Faraday rotation
    - Missing large-scale structure in interferometric data

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   - A smooth distribution of intervening polarized emission
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   - Missing large-scale structure in interferometric data
3. We need to define an invariant quantity under translation and rotation in the Q–U plane.

THE GRADIENT OF LINEAR POLARISATION

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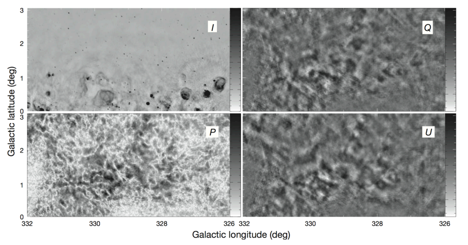
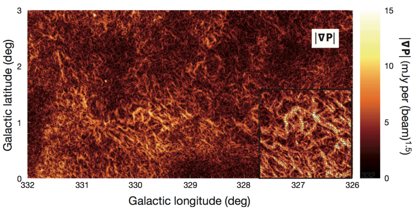
Gaensler et al. 2011

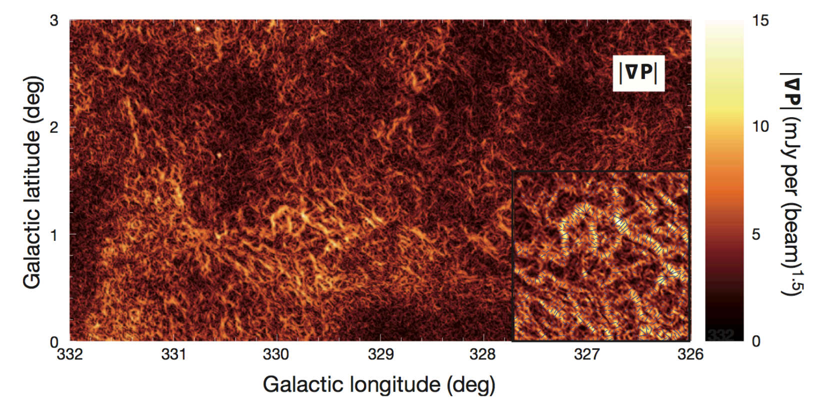




$$|\nabla P| = \sqrt{\left(\frac{\partial Q}{\partial x}\right)^2 + \left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial Q}{\partial y}\right)^2 + \left(\frac{\partial U}{\partial y}\right)^2}$$

$|\nabla P|$ measures the rate at which the polarisation vector traces out a trajectory in the Q–U plane as a function of position on the sky.

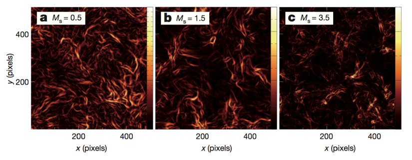
THE GRADIENT OF LINEAR POLARISATION



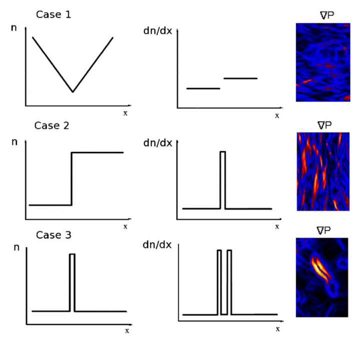
THE GRADIENT OF LINEAR POLARISATION

- Comparisons with MHD simulations
- Define filament types:
  - “Double jumps”
  - Moments (Skewness, Kurtosis)
- Different types of turbulence (subsonic, supersonic)

Gaensler et al. 2011



Burkhart, Lazarian & Gaensler 2012



THE GRADIENT OF LINEAR POLARISATION

$$|\nabla \mathsf{P}| = \sqrt{\left(\frac{\partial \mathsf{Q}}{\partial \mathsf{x}}\right)^2 + \left(\frac{\partial \mathsf{U}}{\partial \mathsf{x}}\right)^2 + \left(\frac{\partial \mathsf{Q}}{\partial \mathsf{y}}\right)^2 + \left(\frac{\partial \mathsf{U}}{\partial \mathsf{y}}\right)^2}$$

- Invariant under rotation and translation in the Q–U plane.
- Trace the rate of change of the polarisation vector in the Q–U plane ($|\mathsf{P}|$ and θ).
- Can reveal properties hidden by a foreground screen (polarised emission or Faraday screen).

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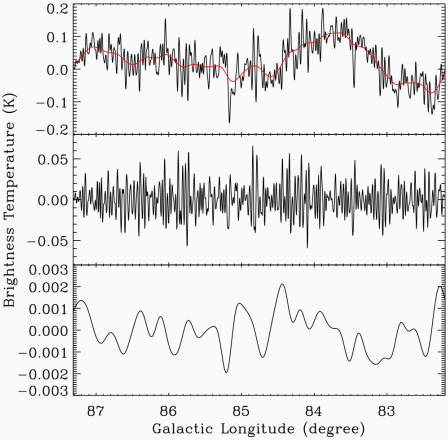
- Trace the rate of change of the polarisation vector in the Q–U plane ($|\mathrm{P}|$ and θ).
- Can reveal properties hidden by a foreground screen (polarised emission or Faraday screen).
- The gradient is only sensitive to the smallest spatial scale.
    - May enhance noise present in the data (Burkhart, Lazarian & Gaensler 2012).

GRADIENT OF STOKES Q

Figure

- Smoothed and unsmoothed signal
- Derivative of the unsmoothed signal
- Derivative of the smoothed signal

Robitaille & Scaife 2015 MNRAS, **451 (1)**, 4891



MULTISCALE ANALYSIS OF THE GRADIENT OF LINEAR POLARISATION

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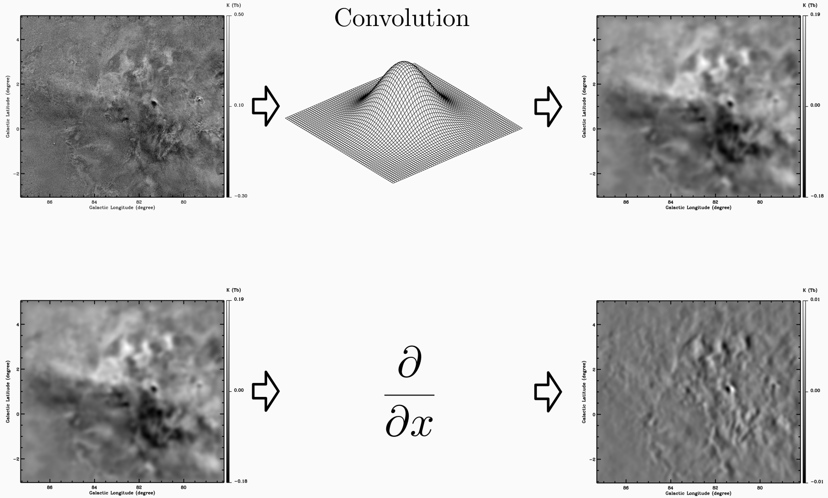

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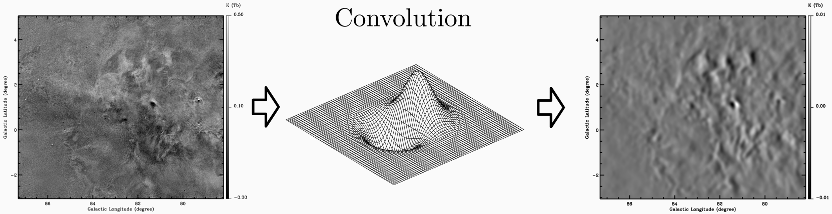


- The direct convolution of the Derivative of a Gaussian (DoG)
- The function satisfies the properties of a wavelet transform

Wavelet Transform

$$\tilde{f}(l, x) = \begin{cases} \tilde{f}_1 = \frac{1}{l^2} \int \psi_1 \left[\frac{(x' - x)}{l} \right] f(x') d^2 x' \\ \\ \tilde{f}_2 = \frac{1}{l^2} \int \psi_2 \left[\frac{(x' - x)}{l} \right] f(x') d^2 x', \end{cases}$$

where

$$\psi_1(x, y) = \frac{\partial^m \phi(x, y)}{\partial x^m} \qquad \text{and} \qquad \psi_2(x, y) = \frac{\partial^m \phi(x, y)}{\partial y^m}.$$

MULTISCALE ANALYSIS OF THE GRADIENT OF LINEAR POLARISATION

Original Gradient of Linear Polarisation:

$$|\nabla P| = \sqrt{\left(\frac{\partial Q}{\partial x}\right)^2 + \left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial Q}{\partial y}\right)^2 + \left(\frac{\partial U}{\partial y}\right)^2}$$

Multiscaled Gradient of Linear Polarisation:

$$|\nabla \tilde{P}(l, x)| = \sqrt{|\tilde{Q}(l, x)|^2 + |\tilde{U}(l, x)|^2},$$

$$\begin{aligned} |\tilde{Q}(l, x)| &= \sqrt{|\tilde{Q}_1(l, x)|^2 + |\tilde{Q}_2(l, x)|^2}, \\ |\tilde{U}(l, x)| &= \sqrt{|\tilde{U}_1(l, x)|^2 + |\tilde{U}_2(l, x)|^2}. \end{aligned}$$

CANADIAN GALACTIC PLANE SURVEY 1.4 GHZ (LANDECKER ET AL. 2010)

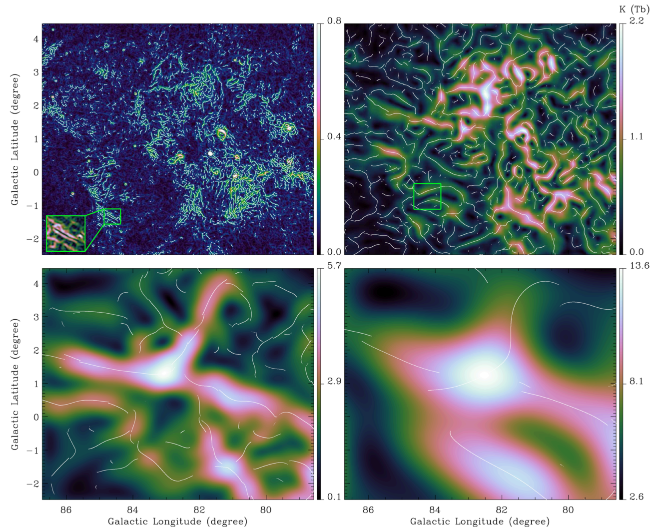


Figure: From left to right: the $|\nabla \tilde{P}(l, x)|$ values at four different scales l= 9.6, 45.7, 153.6 and 434.4 arcmin. White lines represent maxima chains corresponding to the scale (WTMM, Arnéodo et al. 2000, European J. Phys. B, **15**, 567).

POWER SPECTRUM

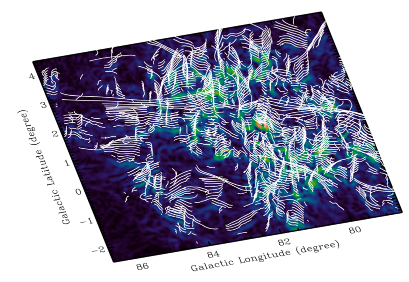
- The power spectrum of an image can be calculated from its wavelet coefficients.
- Δ-variance analysis (Stutzki et al. 1998, Bensch et al. 2001, Ossenkopf et al. 2008)
- Some directional wavelets can reproduce the classical Fourier power spectrum (Kirby 2005, Robitaille, Joncas & Miville-Deschênes 2014).

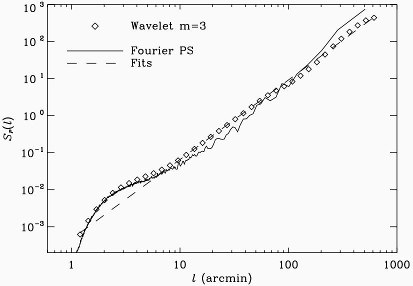
$$\int |f(\mathbf{x})|^2 d^2\mathbf{x} = C_\psi^{-1} \int \int \frac{|\tilde{f}(l,\mathbf{x})|^2}{l^2} dl d^2\mathbf{x} \tag{1}$$

$$E(l) = \int \frac{|\tilde{f}(l,\mathbf{x})|^2}{l^2} d^2\mathbf{x} \tag{2}$$

$$S_P(l) = \frac{1}{N_x N_y} \sum_{\mathbf{x}} |\nabla \tilde{P}(l,\mathbf{x})|^2 \tag{3}$$

Comparison between the wavelet power spectrum of $|\nabla \tilde{P}|$ and the Fourier power spectrum of $|P|$





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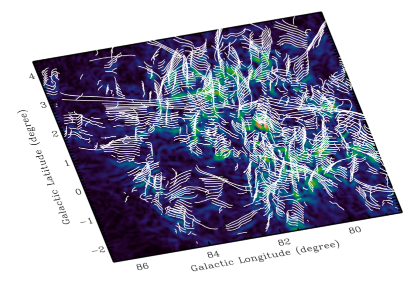
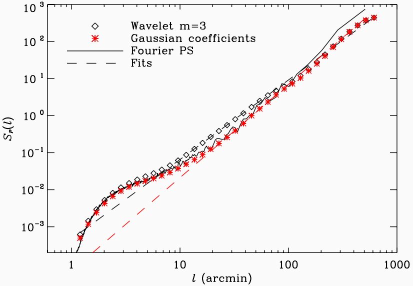




Comparison between the wavelet power spectrum of $|\nabla\tilde{\mathbf{P}}|$ and the Fourier power spectrum of $|\mathbf{P}|$

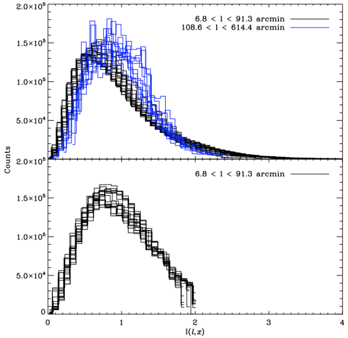
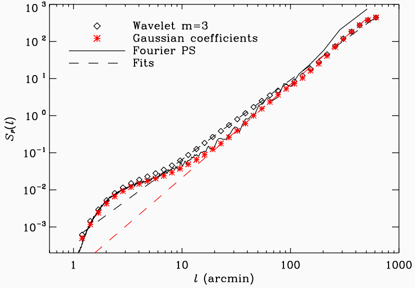


Figure 12. Normalised distributions of wavelet coefficients of $|\nabla\tilde{P}(l,\boldsymbol{x})|$ for the CGPS field for all coefficients (top panel). The black lines represent scales between 6.8 and 91.3 arcmin and the blue lines present scales between 108.6 and 614.4 arcmin. The lower panel shows the Gaussian part of the distribution for scales between 6.8 and 91.3 arcmin.



SYNCHROTRON POLARISATION SEEN IN E & B MODE

Spin ± 2 spherical harmonic decomposition of Stokes parameters Q and U in two opposite parities, the magnetic-type parity (B-modes) and the electric-type parity (E-modes) (Zaldarriaga & Seljak 1997).

- Rotationally invariant quantities.
- The construction of E and B out of Q and U is by its very nature nonlocal.
- Transformation in the spherical harmonic domain.

$$(Q \pm iU)' = \exp(\mp 2i\theta)(Q \pm iU)$$

$$a_{E,\ell m} = -(a_{+2,\ell m} + a_{-2,\ell m})/2$$

$$a_{B,\ell m} = i(a_{+2,\ell m} - a_{-2,\ell m})/2$$

SPIN-2 DECOMPOSITION

- The scalar field E remains unchanged under parity transformation.
- The pseudo-scalar field B changes sign under parity transformation.
- Finding ordered magnetic field at small-scales?

Zaldarriaga 2001 PhRvD, 64 (1), 103001

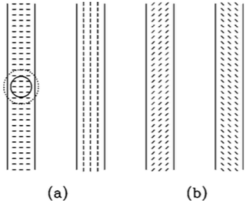


FIG. 3. Examples of polarization vectors inside filaments. The two circles indicate points that contribute with equal weights to *E* and *B* at the center of those circles. The contributions from points along the smaller circle cancel as one moves along the circle. The contribution from the second circle is different from zero. In the case shown the contribution is mainly *E*. The first two filaments [labeled (a)] produce mainly *E* type polarization inside the filaments, while the second two [labeled (b)], produce mainly *B* type polarization.

CONCLUSION

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- Fluctuations traced by $|\nabla\tilde{P}|$ exist at larger scales on data completed with lower spatial frequencies.
- We can measure the power spectrum of $|\nabla\tilde{P}|$ using the wavelet formalism.
- E- & B-mode decomposition of polarised synchrotron emission can revealed ordered magnetic field structures.
- $|\nabla\tilde{P}|$ and E,B-mode decomposition can be used as complementary tools in order to understand complex physical processes involving magnetic fields.
- Future: comparison with simulations (multiple energy-injection scales?)

Questions?