# Notes on POL2 noise and integration times

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## 1 Introduction

The integration time calculator (ITC) for POL2 has as input for the calculation the requested noise level in the Q and U maps. However, the Q and U maps are not the normal end products of the data reduction. The end product is typically a polarization vector map in percentage of the total intensity. To covert the Q and U maps to a fractional polarization map, a total intensity map is required. The total intensity map is also needed for removing the instrumental polarization (IP) from the Q and U maps. Currently the total intensity map is not obtained during a POL2 map - it needs to be obtained separately. The computation of the polarized intensity, vector directions and the removal of the IP effects the noise level in the final result and rises questions like: What is the implication for the required noise level in the Q and U maps? What is the required noise level in the total intensity map? Should the total intensity map be obtained close in time to the polarization map as a part of the program or can maps from SCUBA2 archives be used?

## 2 Noise Considerations

We will only discuss the noise at the 850 $\mu$m band here. The 450 $\mu$m band will be covered when it is commissioned. Note that the IP level at 450 $\mu$m is about 3 % and the polarimeter efficiency $\epsilon$ at 450 $\mu$m is 1/2 compared to 1/1.35 at 850 $\mu$m.

### 2.1 Noise from IP removal

The POL2 instrumental polarization on the JCMT is 1.2 to 1.5 % at 850 $\mu$m. During the IP correction the appropriate fraction of the total intensity map is subtracted from the Q and U maps. Thus at 850 $\mu$m up to 1.5 % of the noise in the total intensity map is added to the Q and U maps during the IP correction. The noise in the corrected map is at most

$$(\delta Q_{corr})^2 = (\delta Q)^2 + (0.015\delta I)^2$$

Using the estimate that the $NEFD$ for the polarimeter is $\sqrt{8}/\epsilon$ times the corresponding SCUBA2 $NEFD$ we have

$$(\delta Q_{corr})^2 = (NEFD_{SCUBA2})^2(\frac{8 \cdot 1.35^2}{T_{POL2}} + \frac{0.015^2}{T_{SCUBA2}})$$

where $T_{POL2}$ and $T_{SCUBA2}$ are the integration times for POL2 and SCUBA2, respectively. Minimizing we find that

$$T_{POL2} = 0.996 \cdot T \quad T_{SCUBA2} = 0.004 \cdot T \quad T = T_{POL2} + T_{SCUBA2}$$

Hence considering only the IP removal it is optimal to spend about 0.5 % of the time on the total intensity map.

## 2.2 Noise in P/I maps

We are only discussing the noise implications - for calibration considerations see below. First note that the noise in the polarized intensity map (P) not is uniform even if the noise level in the Q and U maps is uniform. The noise in the P map is equal to the noise in the Q (or U) map in the regions where polarized intensity is at lest twice the noise level ($\sigma$) in the Q or U maps. In regions of the P map were the polarized intensity is less the noise level is reduced. For a explanation see appendix A.

Let's now compute the noise in the P/I map. We do that by considering noise propagation i.e. consider the effect when a small perturbation is added. Noise propagation is a reasonable approximation when the emission is at least twice the noise in the total intensity map, otherwise this is an underestimate - see discussion in appendix B. Taking the derivative of P/I and divide by P/I we have

$$\frac{\delta(P/I)}{P/I} = \frac{\delta P}{P} - \frac{\delta I}{I}$$

Since typically only the magnitude of the errors are know we have to flip the sign to a plus to estimate an upper limit to the errors. Assuming the errors are normal distributed we will go one step further and add the relative uncertainties in quadrature.

$$(\frac{\delta(P/I)}{P/I})^2 = (\frac{\delta P}{P})^2 + (\frac{\delta I}{I})^2$$

As above we will use $\delta P = 1.35 \cdot \sqrt{8} \cdot NEFD_{SCUBA2}/T_{POL2}$ . Further we assume $I > 10 \cdot P$ so we conservatively can use $I = 10 \cdot P$ . We then get

$$(\frac{\delta(P/I)}{P/I})^2 = (\frac{NEFD_{SCUBA2}}{P})^2 (\frac{8 \cdot 1.35^2}{T_{POL2}} + \frac{1}{10^2 \cdot T_{SCUBA2}})$$

Minimizing for the noise in the P/I map we find that we should spend about 3 % of the total time on the SCUBA2 map and 97 % on the POL2 map. Since both the factors discussed adds noise it is recommended to spend about 5 % of the total time on the SCUBA2 map from noise considerations.

## 2.3 Noise in the vector angle

The vector angle is calculated using $\theta = 1/2 \cdot arctan(U/Q)$. Assuming we have a P signal of at least twice the noise level in the Q (or U) map we can use noise propagation to estimate the angle error.

$$\delta\theta = 1/2 \cdot (\frac{1}{1 + (U/Q)^2} \frac{\delta U}{Q} - \frac{1}{1 + (U/Q)^2} \frac{U \cdot \delta Q}{Q^2})$$

If we again assume normal noise and add the two terms in quadrature this can be simplified to

$$\delta\theta = 1/2 \cdot \frac{\delta Q}{P}$$

where we have assumed $\delta Q = \delta U$. Note the restriction of P being at least twice the noise level in the Q (or U) map for the noise propagation method to be valid. The errors will be larger than indicated by the above expression if P is smaller - see appendix B.

# 3 Calibration

The noise is not the only factor that affects the error in the P/I map. Calibration errors can cause systematic errors in the P/I map. If the interest mostly is in the morphology of the polarization (B-field) this might not be of great importance and using any reasonable total intensity map is acceptable. However, the data can also be affected by issues like bowling around bright sources and resolving out of extended emission. These effects can occur both in the POL2 and SCUBA2 maps but bowling is more prevalent in SCUBA2 maps. So long the ratio between Q and U not is affected i.e. so long an equal fraction of Q and U are resolved out the vector direction is not changed. An equal fractional change is an reasonable assumption. Thus, the main effect of bowling and resolving out emission is presumable errors in the polarization fraction and missing vectors. A decreased amplitude in the P and/or I maps can generate regions of missing vectors due to low S/N. Missing vectors affect the study of the morphology of the polarization. Of these reasons there is a preference for obtaining a SCUBA2 map with a Daisy pattern since the POL2 data is obtained with a Daisy scan. However, tests have not shown any clear difference between using total intensity maps obtained by Daisy or Pong maps in this respect.

If the polarization degree is of importance as in studies of grain properties more care has to be taken. What can cause calibration errors? There are several factors - the FCFs can change slightly with time. Even if the observatory tries to keep on the top of any changes this introduces an uncertainty. The FCF being mainly an optical factor is also effected by thermal gradients distorting the dish. Note that the temperature gradients driven by diurnal temperature changes dominate the FCF variation even if there is a small absolute temperature dependence. Thus the calibration is not necessarily better if the POL2 and SCUBA2 data is obtained the same night - they should also be obtained during similar thermal conditions. Another issue is the the opacity correction. The water vapor radiometer used to estimate the opacity has errors that can vary between different nights or part of nights. The water vapor radiometer measures the brightness temperature on the slope of in the 183 GHz water line and the measurements have errors due to variations in the calibration loads . Further, the conversion from brightness temperature to opacity depends on the atmospheric temperature and humidity profiles while the conversion in done assuming an average atmospheric profile. Thus the conversion is correct in average but has errors due to deviations from the standard atmospheric profile . The amount of seeing effect the effective beam size and perhaps more important introduce larger errors in pointing and focus determinations.

# 4 Recommendations

- It is recommended that the length of the SCUBA2 total intensity map should be approximately 5 % of the time spent on the corresponding POL2 map. Spending more than 10% of the program time on the total intensity map is excessive from noise considerations but might be motivated by calibration concern.

- For time estimates be aware that that at least a 2 $\sigma$, preferable 3 $\sigma$, signal is needed in the P and I maps for reliable signal detection in P and P/I vector maps. If the noise probability distribution have non Gaussian tails i.e. the probability goes down slower than $e^{-x^2/2\sigma^2}$ a higher S/N than 3 is needed for a secure detection. For the P maps the $\sigma$ referred to is the noise level in the Q and U maps - not the noise level in empty sections of the P map. For the total intensity map the $\sigma$ referred to is the noise level in the total intensity map itself.
- Archive SCUBA2 data is fine to use for the intensity map particular if the field morphology is the only science objective.
- Even if SCUBA2 archive data exists and the main interest is field morphology we recommend to include some SCUBA2 total intensity mapping. As a check of the calibration but also to increase the value of the archived data. Other use of the data might have more stringent calibration requirements.
- If the calibration of the fractional polarization is of high importance SCUBA2 data should be obtained just before and/or after the POL2 observations. This can be done by including the SCUBA2 and POL2 observations in the same MSB. A note in the OT program could also be used to request this but there is no guarantee that it will be implemented.
- It is important to take care in the data reduction to minimize bowling and to understand the differences in sensitivity to large scale structures between the POL2 and SCUBA2 maps. Consider spatial filtering to ensure the data have the same sensitivity to large scale structures.

# A Noise in the P map

Assuming the noise in $Q$ and $U$ is normal distributed as $N(\nu\cos(\theta), \sigma_0^2)$ and $N(\nu\sin(\theta), \sigma_0^2)$, respectively, where $\theta$ can take any value. The degree of polarization is then $\sqrt{(\nu\cos(\theta))^2 + (\nu\sin(\theta))^2} = \nu$ and the noise is Rice distributed $R(\nu, \sigma_0^2)$. For $\nu = 0$ this is identical to the Rayleigh distribution fundamental to radio astronomy. The Rice distribution is plotted in Figure 1 for different values of $\nu/\sigma_0$.

Above $\nu = 2$ the distribution is close to the normal distribution $N(\nu, \sigma^2)$ while below $\nu = 2$ the expectation value of the distribution is noticeable higher than $\nu$ and the variance is smaller than $\sigma^2$. This can be seen in figure 2. The need to de-bias P maps is due to the difference between $\mu$ and $\nu$.

# B Noise in P/I and vector angle

$P/I$ and $Q/U$ are not well behaved when there is no signal in the I or U maps - due to the division by noise with an average value close to zero. The ratio between two zero mean normal distributed variables, as is the case for $Q/U$ when P is zero, have a Cauchy distribution. This distribution has no expectation or variance - all moments from the expectation value and up diverges. Which make sense since we not can define a polarization angle or error if there is no polarization! The limiting action of the *arctan* function reduces the error but the error is still $\pm\pi/2$ for no polarization signal. The P/I division has the same properties but is not limited by an *arctan* function and there is no

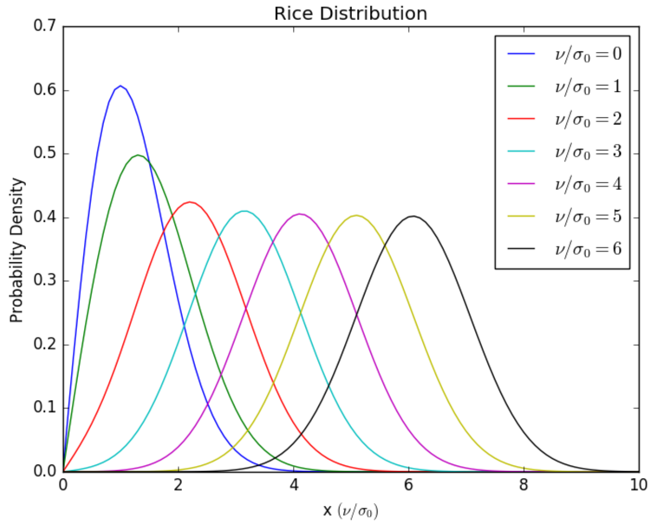


Figure 1: Rice distribution for different values of the signal $\nu$ measured in units of $\sigma_0$. The independent variable x is in units of $\nu/\sigma_0$

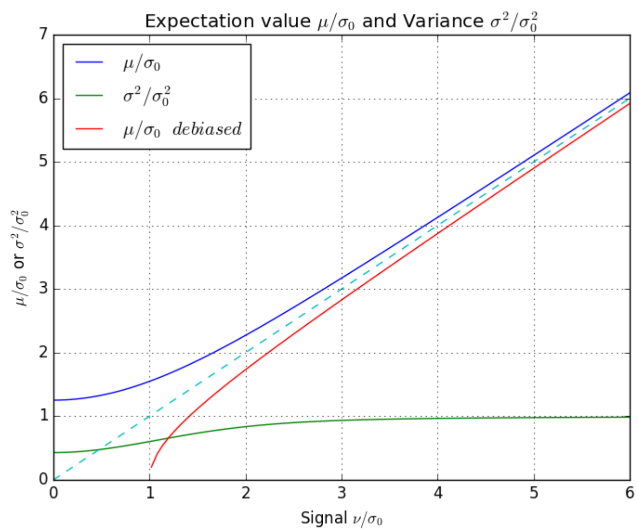


Figure 2: Expectation values of $\mu$, de-biased $\mu$ ($\sqrt{\mu^2 - \sigma_0^2}$) both in units of $\sigma_0$ and variance $\sigma^2/\sigma_0^2$ as function of signal level $\nu/\sigma_0$ where $\sigma_0^2$ is the variance of the normal distributions the Rice distribution originated form

limit to the errors. When a expectation value different from zero is introduced in the denominator the errors starts to go down and are well described by error propagation above a signal level of about 2 $\sigma$. However, using a limit of at least 3 $\sigma$ is recommended. At 2 $\sigma$ there is still a significant chance to divide by a number close to zero due to noise which will introduce a large deviant error at that point. This is even more the case if the noise distribution has non Gaussian behavior.